2.3.1 INTRODUCTION: LANCZOS'S CONTRIBUTIONS TO RELATIVISTIC QUANTUM MECHANICS

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In February 1928, Dirac discovered the relativistic wave mechanical equation[1] for the electron described as "one of the marvels of twentieth century physics."[2] Dirac's equation involved a four-component spinor wave function which correctly describes in a natural manner the electron's spin and magnetic moment and the fine structure in the hydrogen spectrum.[3] Nonetheless, in spite of the excitement associated with these successes, Dirac's relativistic equation for the electron initially led to problems, frustration and doubts in the minds of most of the founders of nonrelativistic quantum mechanics. For Lanczos, however, Dirac's discovery revived his interest in quantum theory and he returned to the problem of obtaining a field-like formulation of quantum theory as characterized in the introduction to Section 2.2.1.

This section is devoted to a discussion of Lanczos's relativistic quantum papers[4] (Lanczos 1929b,c,d 1930a) and includes commentaries by André Gsponer and J.-P. Hurni, and J. R. McConnell.

Lanczos's first paper which reformulated Dirac's equation in terms of quaternions (Lanczos 1929b) was received 17 July 1929. During 1929, he submitted three additional papers on this subject for publication (Lanczos 1929c,d 1930a). In connection with this work, it is of interest that Lanczos mentioned in his paper 1929b that he had become proficient in dealing with quaternions in the context of Minkowski space-time ten years earlier while reformulating Maxwell's equations in the framework of biquaternion analysis as a part of the research for his dissertation. In the course of this work, he had found equations of the form of Dirac's equations without the mass term.[5]

Lanczos's renewed hopes of finding a fundamental field-like formulation for quantum theory probably first peaked during 1929, the year he was awarded a fellowship by the *Notgemeinshaft der Deutschen Wissenschaft* to spend a year in Berlin assisting Einstein with mathematical work. There is no information that indicates whether Lanczos and Einstein discussed these or any subsequent papers dealing with Dirac's equations. It would be very surprising, indeed, if Einstein and Lanczos never mentioned Dirac's equation and its implications. During 1929, we know that much of Einstein's attention was focused on his attempt to formulate a unified field theory connected with the concept of "distant parallelism"[6] (later abandoned), a theory that apparently had little appeal for Lanczos.[7]

It is clear that during this period the viewpoints and interests of Lanczos and Einstein differed to some extent as evidenced by the works of Lanczos relating to Dirac's theory. Certainly, these works of Lanczos relating to Dirac's theory were done in competition with many other activities, duties, and interests. In this section, we will attempt to understand the thoughts and motivations of Lanczos in

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[1] P.A.M. Dirac, *Proceedings of the Royal Society*, A **117** (1928): 610.

[2] See the book by A. Pais, *Niels Bohr's Times, in Physics, Philosophy, and Polity* (Pais, 1991, 352).

[3] It will be recalled that in 1927 Pauli (*Zeitschrift für Physik*, **43** [1927]: 601) had made the ad hoc proposal to describe the spin of the electron in non-relativistic quantum theory by means of a two component wave function.

[4] C. Lanczos, *Zeitschrift für Physik*, **57** (1929): 447, here referred to as (Lanczos 1929a); **57** (1929): 474, here referred to as (Lanczos 1929b); (1929): 484, here referred to as (Lanczos 1929c); *Physicalische Zeitschrift*, **31** (1930): 120, here referred to as (Lanczos 1930a), and published form of an invited paper given 25 October 1929 at a meeting of the Berlin Physical Society.

[5] In connection with the content of Lanczos's dissertation, see the commentary by A. Gsponer and J.-P. Hurni in Section 2.1.

[6] A. Einstein, *Preuss. Acad. Wiss. Sitz.* (1928): 217, 224; (1929): 156; (1930): 18; *Math. Ann.* **102** (1930): 685.

[7] C. Lanczos, *Ergebnisse der Exakten Naturwissenschaften*, **10** (1931): 97–132.

writing these papers briefly by first reviewing the state of quantum theory during the period 1928–1929 and then considering the things we now know and also the important questions we cannot answer.

By 1928 non-relativistic quantum mechanics was in good shape and making very important progress the in areas of atomic and solid state physics. However, some difficulties had already been encountered in the early stages of the development of quantum electrodynamics. Then with Dirac's 1928 paper came the discovery of the relativistic wave mechanical equation for the electron. However, for all the successes of the Dirac theory, it seemed that new difficulties were being revealed nearly every day in this early period. Everyone, including Dirac, wondered how a theory could be so successful and yet so seemingly full of paradoxes. In a letter to Pauli dated 31 July 1928, Heisenberg writes:

> The saddest chapter of modern physics is and remains Dirac's theory (Pais 1986, 348).

Much later Heisenberg described the situation in this early period as follows:

> Up till that time I had the impression that in quantum theory we had come back into port. Dirac's paper threw us out into the sea again (Pais 1986, 347).

The situation remained essentially unchanged until December 1929 when Dirac had a new idea[8] about how to deal with the negative energy states in terms of his "hole theory," which led to the prediction of the positron[9] in May 1931. Most of the reactions to the hole theory were negative, even after Anderson's discovery of the positron in cosmic ray experiments.[10] Bohr and others were slow to accept Dirac's theory. For example, Pauli wrote to Dirac in a letter dated 1 May 1933:

> I do not believe in your perception of "holes" even if the "anti-electron" is proved (Pais 1986, 360).

It was soon recognized that quantum electrodynamics and the quantized field theory for Dirac's equation required fundamental changes in the treatment of particles and fields and their interactions in order to satisfy the Pauli exclusion principle.

Dirac's theoretical work provided the first example in which quantum theory, on purely theoretical grounds, had predicted the existence of a new particle. He next predicted the existence of "negative protons" during his Nobel lecture on 12 December 1933.

By the time Lanczos presented an invited paper on Dirac's wave mechanical theory of the electron on 25 October 1929 at a meeting of the Berlin Physical Society, he had published three papers (Lanczos 1929b,c,d) relating to Dirac's theory. At the beginning of this talk (Lanczos 1930a), he presented a discussion of Dirac's theory that contained a number of interesting analogies and insights. However, Lanczos's discussion did not contain any remarks about such matters as the negative energy states or the Klein paradox.[11] On the other hand, when shifting over to his own work, Lanczos clearly expressed the idea that the problem he saw in Dirac's theory was of a much more fundamental nature:

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[8] P.A.M. Dirac, *Proceedings of the Royal Society*, A **126** (1929): 360.

[9] P.A.M. Dirac, *Proceedings of the Royal Society*, A **133** (1931): 60.

[10] It has been pointed out that C. D. Anderson did not know about Dirac's theory when he discovered the new particle in the cloud chamber (*Phys. Rev.*, **43** [1933]: 492) (Mehra 1973, 58). In this connection, A. Pais in his book (Pais 1991, 352) offers the following remarks attributed to C. D. Anderson: "Yes I knew about the Dirac theory. . . . But I was not familiar in detail with Dirac's work. I was too busy operating this piece of equipment to have much time to read his papers. . . . [Their] highly esoteric character was apparently not in tune with most of the scientific thinking of the day. . . . The discovery of the positron was wholly accidental." This paper by Anderson was submitted in September 1932.

[11] O. Klein submitted a paper (*Zeitschrift für Physik*, **53** [1929]: 157) in December 1928 which appeared to confront Dirac's theory with disastrous consequences. Namely, when a beam impinges on a potential barrier that varies by more

> Ich habe den Versuch gemacht — bzw. eine gelegentliche Beobachtung hat mich dazu gefürt — diese Theorie so zu ergänzen, daßsie einen wirklichen feldtheoretischen Charakter bekommt. (Lanczos 130a, 123)
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> [I have made the attempt—better to say a chance [gelegentliche] observation led me—to enlarge the [Dirac's] theory in such a way that it obtains a true field theoretic character.]

Lanczos's observation was the fact that Dirac's equation could be *derived* from a more fundamental system which is formally very similar to Maxwell's equation, except for a supplementary term containing the mass. If this equation is written in quaternion notation, it takes a particularly simple form. Moreover, since Maxwell's theory has an unambiguous tensorial interpretation,[12] Lanczos was led to think that he had found a "*via regia* to grasp the internal substance of Dirac's equation" (Lanczos 1929b, p. 449).

With considerable assurance, and in his very characteristic didactic style, Lanczos passionately writes down his derivation of Dirac's equation based on quaternionic theory. He produces a very modern article (Lanczos 1929b), which over sixty years later still contains a number of ideas which remain at the forefront of fundamental theory. He displays a mastery of quaternionic algebraic techniques and, with a very careful choice of notations and minimal usage of redundant mathematical jargon, he ends up with a physics paper so different in style from those of his time, that one could think that he wrote it for the future. Indeed, using that formalism, he showed that Dirac's equation could be derived from a more fundamental quaternionic system, provided one accepts a *doubling* which today we recognize as *isospin*. Then, despite various problems of interpretation, Lanczos discovers that in addition to Dirac's equation, his fundamental system leads to another fully consistent set of wave equations. In fact, he found, seven years before Proca,[13] the correct form of the wave function for a spin one particle.[14]

In the third paper of this series (Lanczos 1929d), Lanczos established that this spin one equation has a proper energy-momentum conservation expression and tried to apply it to the electron where he was led to introduce the notion of a variable mass. In the fourth paper in this series (Lanczos 1930a), after briefly reviewing Dirac's theory and his own work (Lanczos 1929b,c,d), Lanczos proceeded to argue that his version of a non-linear electron theory with variable mass could be naturally formulated in a conformally flat space-time. Then he proceeded to discuss briefly the possible form of the solutions to the new field equations and concluded:

> So ist es also offenbar noch verfrüht, ein so tiefliegendes Problem, wie die Struktur des Elektrons, mit diesen elementaren Ansätzen in Angriff zu nehmen. Dennoch bin ich des Glaubens, daß die im Vorangegangenen skizzierte Untersuchung, die formale Beziehungen der Diracshen Theorie zu den Grundgleichungen des elektromagnetischen Feldes hinausgehend noch etwas Tiefergreifendes darstellt; nämlich einen ersten, wenn auch noch weitgehend unvolkommenen Versuch, die Quantenprobleme gemeinsam mit dem Problem der

than mc^2, where m is the mass of the electron, Klein showed that Dirac's theory predicts that more electrons are emitted by the barrier than impinge on the barrier. It is well-known that this seeming paradox was later explained in terms of electron-positron pair production.

[12] Like many other physicists of the time, Lanczos was puzzled by the fact that the Dirac wave-function for the electron did not have an ordinary tensor character. As we now know, the transformation properties of the Dirac field are those of a spinor.

[13] A. Proca, *J. Phys. Radium*, 7 (1936): 347-353.

[14] Of course, Lanczos was not aware that he had discovered the correct equation for a spin one particle. He simply assumed that all covariant solutions of his fundamental system would in some way apply to the electron, while in fact they correspond to elementary particles of different intrinsic spin.

Materie von einer einheitlichen feldtheoretischen Basis aus zu verstehen.(Lanczos 1930a, 130)

[It is clearly still too early to tackle a problem as deep as the structure of the electron with these elementary assumptions [Anzatäzen]. Nevertheless, I am of the belief that the investigation outlined above, which reveals formal relations of the Dirac theory to the equations of the electromagnetic field, goes beyond the purely formal to represent something deeper; namely, a first step, though by far incomplete, toward an understanding of the quantum problems and together with the problem of matter, on a unified field theoretical basis.]

As already indicated, Lanczos wrote the four papers (Lanczos 1929a,b,c, 1930a) relating to Dirac's relativistic quantum theory while working as Einstein's research assistant in Berlin. These papers on Dirac's theory have no acknowledgments and it appears that Einstein had little or no interest in them. However, about three years later, Einstein and Mayer published a derivation of Lanczos's version of Dirac's equation, using their semivector formalism, which has many strong similarities with Lanczos's work.[15]

In the perspective of the contemporary significance of Lanczos's papers on Dirac's equation, and more particularly of Lanczos's discovery that Dirac's equation can be derived from a more fundamental system provided one accepts a doubling, it is important to remember that experiment has shown that the most elementary particles of matter—the lepton and the quarks—are indeed associated in pairs. In fact, a careful analysis, which was done for the first time by Gürsey in 1957 for the case of the proton-neutron doublets,[16] shows that the doubling discovered by Lanczos is exactly of the kind that is required for the isospin interpretation of these pairs. Since Proca's equation can also be derived from the same fundamental system, Lanczos's discovery may therefore be more than just a mathematical artifact, in which case the more fundamental fields introduced by Lanczos should have some profound physical significance.[17]

During the period 1929-1931, it appears that Lanczos's papers were ignored by the leading researchers in the field. This can be explained in part by the relative success as well as the problems of the standard Dirac theory, which did not seem to require a generalization of the kind Lanczos's theory was implying.

The considerations of this section cannot even remotely approach providing a complete understanding of Lanczos's thoughts at this time concerning the implications of Dirac's relativistic wave equation for the electron and his attempts to make contributions in this area. Nonetheless, a number of interesting questions arise concerning the path that Lanczos followed during the 1930's. In particular, the following four questions give some measure of the extent of our incomplete knowledge of his motivations and the ideas behind Lanczos's attempts to contribute to relativistic quantum theory:

(1) Given Lanczos's deep interest in relativity theory and field theory and his idea that quantum mechanics should be interpreted as a field theory, why did he appear to show so little interest in field quantization (or second quantization) and the development of quantum electrodynamics?

(2) Considering the fact that Lanczos worked with Einstein during 1929 on the theory of "distant par-

[15] A. Einstein and W. Mayer, *Proc. Roy. Acad. Amsterdam*, **36** (1933): 497-516, 615-619.

[16] F. Gürsey, *Il Nuovo Cimento*, 7 (1958): 411-415.

[17] The possibility of these and other implications of Lanczos's work being relevant to modern research are discussed in the commentary of A. Gsponer and J.-P. Hurni.

allelism," which, together with his own work on Dirac's theory, certainly prepared him to deal with tetrad formalism and thus with spinor formalism in the context of general relativity, why did he give no evidence of interest in works of Fock[18] and Weyl[19] (1929) on the covariant formulation of Dirac's wave equation for the electron and its generalization[20] in the framework of Einstein gravitational theory?[21]

(3) Why did the discovery of the positron (i.e., the first example of antimatter) not provide new stimulus for Lanczos to make further speculations in regard to Dirac's theory as a fundamental field theory connected with Einstein's gravitational theory or some immediate generalization thereof?

(4) What were Lanczos's thoughts about the quantum riddle and the problem of matter?[22]

We have very little information to guide us in any type of attempt to try to answer these questions. Therefore, we make no serious general attempt to do so.[23] Nonetheless, we want to introduce the one important piece of definite information we have that certainly has some bearing on these questions; namely, that by 1933 Lanczos had become convinced that the deeper mathematical structure under-

[18]V. Fock, *Zeitschrift für Physik*, 57 (1929): 261, V. Fock and L. Infeld, *Comptes Rendus*, 188 (1929): 1470, and V. Fock, *Comptes Rendus*, 189 (1929): 25.

[19]H. Weyl, *Zeitschrift für Physik*, 56 (1929): 330

[20]There is some evidence that he read these papers as well as a note by Wiener (N. Wiener, *Nature* 12b [1929]: 330) stressing the physical importance of using the Ricci principal directions in connection with tetrad formalism.

[21]If Lanczos had been able to sustain enough faith in the fruitful consequences of some sort of comprehensive fundamental theory involving Dirac's relativistic wave equation and general relativity, one could imagine him developing, over the years, a theory having some similarities with the work (generally regarded as flawed) Eddington was working on at the time of his death in 1944. Whittaker referred to this work as "Eddington's fundamental theory" (Arthur Eddington, *Fundamental Theory*, E. T. Whittaker, ed., Cambridge Univ. Press, Cambridge, 1946).

[22]Mehra and Rechenberg (Mehra, Rechenberg 1982, 219) make the following interesting statements in connection with this question:

> During his stay in Berlin, Lanczos considered the possibility of deriving the universal quantum function or quantum condition from a nonlinear generalization of the Dirac equation. In August 1929 his hopes of completing *his* field-like representation of quantum mechanics ran high. 'If the possibilities surmised here should be capable of being realized, then quantum mechanics would cease to be an independent discipline.' Lanczos remarked in conclusion of his renewed effort. 'It would fuse with a deepened "theory of matter," which must be built upon regular solutions of nonlinear differential equations, that is, ultimately merge into the "world equations" of the universe. The dualism of "matter and field" would then be overcome just as the dualism of "corpuscle and wave"' [Lanczos 1929c, 493]. Had Lanczos succeeded in deriving the quantum condition from field theory, he would have been able to answer at the same time Einstein's question; namely, whether field theory offered the possibility of solving the quantum riddle, in the affirmative sense (A. Einstein, [*Sitz. Ber. Press. Akad. Wiss.* [1923]: 359–364]). He never did, nor did anyone else after him.

[23]However, here we will attempt to make some brief reasonable speculations about the apparent lack of interest that Lanczos had for quantum field theory. The difficulties of quantum field theory as compared to quantum mechanics were in evidence from the beginnings of quantum electrodynamics. W. Heisenberg (interview with T. S. Kuhn, 25 February 1963, NBA [Pais 1986, 348]) expressed these difficulties with quantum electrodynamics:

> You know, it was not like quantum mechanics, in quantum mechanics everything came out much simpler and much better than what I expected. Somehow when you touched it and you had a disagreeable difficulty, at the end you saw, "Well, was it that simple?" Here in electrodynamics, it didn't become simple. Well, you could do the theory, but still it never became that simple.

These difficulties added to those of classical electrodynamics in trying to account for the electron might explain Lanczos's lack of interest. Some of his motivations to generalize Dirac's theory might be best understood in the framework of the motivations underlying the generalized classical theories proposed by some of the founders of modern quantum theory on the basis of their frustration with the problems of quantum electrodynamics in the 1930's extending well in the 1940's. In this connection consider, for example, the Born–Infeld theory of electrodynamics (*Proceedings of the Royal Society* [London] A 143 [1933]: 410; A 144 [1934]: 425), Dirac's theory of radiating electrons (*Proceedings of the Royal Society* [London] A 167 [1938]: 148), and the generalized electrodynamics of F. Bopp (*Zeitschrift für Physik*, 32 [1948]: 564) and R. P. Feynman (*Phys. Rev.*, 74 [1948]: 939; 1430).

lying Hamiltonian theory was the central key to making progress in quantum theory and field theory. Lanczos indicates that this insight came to him during 1931–1932 when he was preparing lecture notes for graduate courses covering analytic mechanics and advanced quantum mechanics, including Dirac's theory, when he was at Purdue University. In particular, we have the following account by Lanczos in the introduction to his paper:

> Während des letzten Wintersemesters hatte ich an der hiesigen Universität zwei je dreistündige Vorlesungskurse abgehalten: die eine in analytischer Mechanik, die andere in fortgeschrittener Wellenmechanik. Im Laufe dieser Vorlesungen bin ich einerseits mit den bewunderungswürdig schönen Methoden der Hamiltonschen Dynamik in näheren Kontakt gekommen (andererseits habe ich Gelegenheit gefunden, die fundamentale Diracsche Gleichung des elektrons eingehend zu besprechen). Durch die Parallelität der beiden Vorlesungen trat mir eine eigentümliche Korrelation plastisch vor Augen: auf der einen Seite die Lagrangeschen Gleichungen, die von zweiter Ordnung sind und durch die Hamiltonsche Methode linearisiert werden, wobei die Zahl der Veränderlichen sich verdoppelt. Auf der anderen Seite die Schrödingersche Gleichung, auch von zweiter Ordnung, und wiederum ihre Linearisierung durch das Diracsche System, wobei die Variablenzahl sich ebenfalls erhölt. Sollte da ein mehr als bloßformaler Zusammenhang vorliegen? (Sollte es sich um eine wirkliche Identität handeln?) Sollte am Ende die ganze Wellenmechanik eine Anwendung der Hamiltonschen kanonischen Gleichungen sein? Diese Vermutung hat sich in einem Umfange als richtig erwiesen, der alle ursprünglichen Erwartungen des Verfassers übertroffen hat [Lanczos 1933a).
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> [During the winter semester at the above cited university, I held two three-hour lecture courses: one on analytic mechanics, the other on advanced wave mechanics. In the course of these lectures, I came in closer contact with the wonderfully beautiful methods of Hamiltonian Dynamics, and on the other side, I had the opportunity to discuss Dirac's equations for the electron. Through the parallelism of these two lectures a peculiar correlation took place tangibly before my eyes: on the one side stands the Lagrangians that are of second order and linearized by the Hamiltonian methods, with a simultaneous doubling of variables. On the other side, there is the Schrödinger equation, also of second order, together with its linearization by Dirac's system, also with the number of variables increased. Could there be more than merely a formal connection here? Could it have to do with a real identity? In the end, could it be that wave mechanics is nothing more than an application of Hamilton's canonical equations? This conjecture proved its validity to an extent far exceeding all the initial expectations of the author.]

This conviction regarding the fundamental character of the underlying mathematical structure of Hamiltonian theory appears to have grown throughout the 1930's and 1940's as Lanczos attempted to apply these ideas in many areas relating to quantum theory and general relativity. In the next few sections of Part 2, we will consider a number of examples of these ideas of Lanczos.

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