K_S Regeneration on Electrons from 30 to 100 GeV/c: A Measurement of the K^0 Charge Radius

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The amplitude of K_S regeneration by electrons, $f_{21}^e/k = -(\alpha/3)\langle R^2\rangle$ ($\langle R^2\rangle$ is the K^0 charge radius), can be determined by comparing the rates of *coherent* (transmission) regeneration and of *diffraction* regeneration at $q^2=0$. We made a determination from 30 to 100 GeV/c, using a novel approach: Two *distinct* Pb regenerators, of optimized thicknesses, were exposed to a double beam, and interchanged every burst. We find $\langle R^2\rangle = -(0.054\pm0.026)$ fm^2. The sign, magnitude, and p independence agree with predictions.

Twenty years ago, Feinberg[1] and Zel'dovich[2] pointed out that K^0's could be elastically scattered from electrons, with an amplitude f^e proportional to $\langle R^2\rangle$, the mean-square K^0 charge radius. The normalization problem is solved by the double-beam technique,[9,10] i.e., by observing the two butions. In the naive, nonrelativistic quark model $\langle R^2\rangle$ should be finite and negative, since the two ($\bar{s}$ and d) quarks orbit about their center of mass, with the lighter d quark farther out. Quantitative estimates of $\langle R^2\rangle$ have been provided by a number of authors[2-5]; the negative sign survives in a relativistic treatment.

During the past decade, two efforts[6,7] to measure $\langle R^2\rangle$ were made. We report here on the first sensitive measurement; it is based, like Refs. 6 and 7, on the *coherent regeneration of K_S's by atomic electrons*. The principles of this method are due to Zel'dovich.[2] Let us summarize them briefly before discussing our actual approach.

The scattering of K_L's on electrons will lead to K_S regeneration, since $f^e \neq \bar{f}^e$. The amplitude for this process (at K_L momentum $\hbar k$) is characterized by the quantity

$$f_{21}^e/k \equiv (f^e - \bar{f}^e)/2k = -(\alpha/3)\langle R^2\rangle, \tag{1}$$

formulated in analogy to the familar nuclear quantity

$$f_{21}^n(q^2)/k \equiv [f^n(q^2) - \bar{f}^n(q^2)]/2k. \tag{2}$$

Note that (1) is independent of four-momentum transfer q^2 and of k, whereas the amplitudes in (2) depend on both variables. At $q^2=0$, all scattering centers (nuclei and electrons) in a regenerator with N nuclei/cm^3 and of length L will act *coherently*, with an amplitude

$$\rho = 2i\pi NL\left(\frac{f_{21}^n(0)}{k} + \frac{Zf_{21}^e(0)}{k}\right)\frac{\varphi(l)}{l}, \tag{3}$$

where $l = Lm_K\Gamma_S/pc$ and $\varphi(l)/l$ is a known factor[7] for finite L, such that $\varphi(l)/l \to 1$ as $L\to 0$. Coherent (or "transmission") regeneration yields (per K_L) a K_S rate $I_{\rm tr}(0)$ proportional to $|\rho|^2$, or, more accurately, to $|\rho|^2\exp(-L/\lambda)$, where λ is the K_L mean free path in the regenerator material. As $Zf_{21}^e \ll |f_{21}^n|$, coherent regeneration on electrons contributes a "correction" of $2\,\mathrm{Re}[Zf_{21}^e/f_{21}^n(0)]$ to $I_{\rm tr}(0)$. For finite q^2, the centers no longer scatter coherently, and the electron contribution can safely be neglected. In this region, that of "diffraction" regeneration (due to elastic nuclear scatterings), one has ideally (per K_L)[8]

$$\frac{dI_{\rm diff}}{dq^2} = \pi NL\left|\frac{f_{21}^n(q^2)}{k}\right|^2\left(\frac{1-e^{-l}}{l}\right)\exp\left(\frac{-L}{\lambda}\right). \tag{4}$$

By measuring *both* $I_{\rm tr}(0)$ and $dI_{\rm diff}/dq^2$ *extrapolated* to $q^2=0$ one can isolate the "correction" of interest. In particular, for an ideally thin regenerator, one can use the ratio

$$R = \left(\frac{I_{\rm tr}}{dI_{\rm diff}/dq^2}\right)_{q^2=0} = 4\pi NL\left[1 + 2\,\mathrm{Re}\left(\frac{Zf_{21}^e}{f_{21}^n}\right)\right]. \tag{5}$$

The advantage of measuring both $I_{\rm tr}$ and $dI_{\rm diff}/dq^2$

simultaneously with a single regenerator of fixed L, as proposed by Zel'dovich[2] and adopted in Refs. 6 and 7, lies in the fact that this method is self-normalizing; i.e., the attenutated K_L flux drops out in R. The disadvantage is that one has to compromise in the choice of L: One would choose a thick block to maximize the coherent signal and to reduce the correction for CP nonconservation which is goverened by $|\rho/\eta|$, while the idealizations leading to (4) call for a very thin one, in particular to reduce the correction for multiple scattering. This compromise necessitates in practice large corrections and numerous ancillary measurements.

The approach of the present work is to use two Pb regenerators, viz., a thick and a thin one. The normalization problem is solved by the double-beam technique,[9,10] i.e., by observing the two regenerators virtually simultaneously, and by knowing the attenuation (that is, σ_T for K_L on Pb[9]) very precisely. Independently of this approach, the expected effect is considerably enhanced (for given $\langle R^2\rangle$) in our energy range of 30 to 100 GeV/c, since $f^n(0)/k$ falls roughly[10] as $k^{-0.61}$, while f^e/k is constant. At 65 GeV/c, the effect should be about 3% based upon the estimate in Ref. 4.

The experimental setup consisted of a double-regenerator arrangement, a short evacuated decay region ($2\tau_s$ at 50 GeV), and a multiwire-proportional-counter spectrometer.[11] A thin (2 cm) and a thick (28 cm$\simeq 2\lambda$) Pb regenerator were each hit by a separate (5.9×5.9 cm^2) beam of 2.5×10^5 K_L's per burst. The thin one was a plate covering both beams and encased in anticounters, while the thick one consisted of a block ($7.0\times7.0\times26$ cm^3) *and* the plate. The block was placed within a sweeping magnet 40 cm upstream of the plate and provided with a mechanism to move it from one beam to the other between bursts; the roles of the beams were thus interchanged about 2×10^5 times. The exit of the decay region was defined by a 1-mm scintillator framed by four photon counters. We recall that the spectrometer includes, for K_{l3} suppression, a shower counter and a muon hodoscope; here the muon hodoscope was in active veto while the shower-counter pulse heights and the hits in the photon counters were tagged.

Pattern recognition was done on line; about 75% of the 250 triggers per burst reconstructed as V's. Roughly 35×10^6 events were recorded. Off line, only tracks with $p>10$ GeV/c were retained and standard geometrical cuts[11] were applied. To suppress K_{e3}'s events with either track

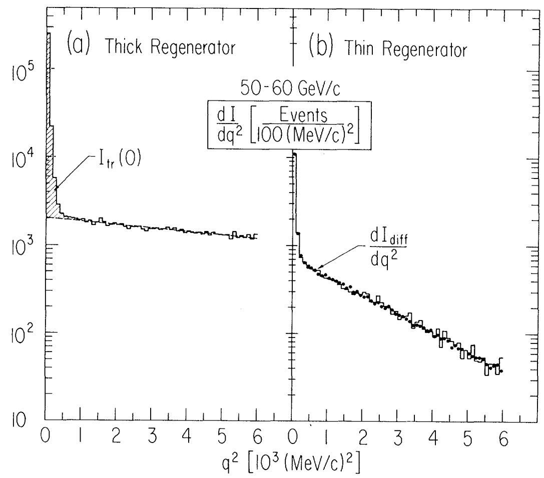

FIG. 1. Acceptance-corrected q^2 distribution of $K_{\pi2}$'s for one momentum bin. The dashed line in (a) is fitted exponential to events with $q^2>700$ (MeV/c)^2, while the dots in (b) represent the multiple-scattering–corrected diffracted events, calculated as indicated in the text, including the coherent peak.

depositing $>0.7pc$ in the shower counters were rejected. A mass cut at 1115 ± 5 MeV eliminated Λ's. About 2×10 $K_{\pi2}$ candidates were thus isolated. Three types of background were to be subtracted: (a) unsuppressed K_{e3}'s (3%), (b) unsuppressed $K_{\mu3}$'s (0.5%), and (c) V's produced in the 125-μm entrance window of the decay pipe. The distribution in $m_{\pi\pi}$ and q^2 for each of these was determined from identified events, and for each of seven p bins of actual data a three-parameter fit outside the m_K peak was performed and the backgrounds subtracted. No other sources of background were found. A cut at $m_K\pm20$ MeV yielded the final q^2 distribution for each regenerator. Figure 1 shows these (acceptance-corrected) distributions for $p=55\pm5$ GeV/c.

For each p bin, the q^2 distribution from the thin regenerator was fitted for $q^2<2\times10^5$ (MeV/c)^2 by the sum of four distributions: (a) a coherent peak (I_{tr}), whose shape was determined from the thick-regenerator data [e.g., Fig. 1(a)] by extrapolating events at finite q^2; (b) a distribution for diffracted events, including first- and second-order multiple-scattering corrections,[12] with amplitudes $f_{21}(q^2)$ and $f_{22}(q^2)$ calculated from an optical model,[10] where f_{22} was calculated with the conventional input parameters constrained by σ_T,[9] while the shape of the f_{21} was taken from the model, with the neutron radius left free in the fit; (c) a distribution of Primakoff-produced $K^*(892)$'s decaying with an undetected π^0; (d) an exponential

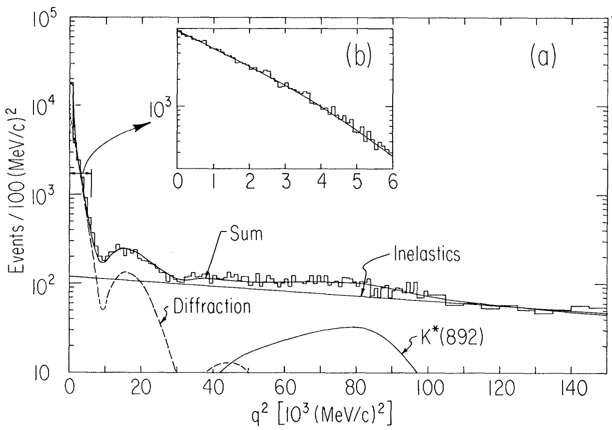

FIG. 2. (a) Acceptance-corrected q^2 distribution of $K_{\pi 2}$'s for all data. The three extracted contributions at $q^2 > 0$ are indicated and labeled. (b) Fit to dI/dq^2 for $q^2 < 6000$ (MeV/c)^2.

distribution of inelastically produced K_S's. The calculated $f_{22}(q^2)$ was accurately checked by measuring R for the thick regenerator, and the shape of (d) agreed with an independent determination from thick-regenerator events in which the anticounter just upstream of the thin regenerator fired. Figure 2 shows the final fit to all of the data; it is seen to be good over the entire q^2 range. The clear emergence of the two diffraction dips,[13] largely filled in by inelastic events, is strong evidence that the contribution of the latter at $q^2 = 0$ (1.5%) is being correctly determined. With (c) and (d) determined, their contributions are constrained in a new fit to the events with $q^2 < 6000$ (MeV/c)^2. Figure 2(b) displays this fit, with the coherent signal subtracted. It is evidently necessary to extrapolate to $q^2 = 0$ with a Bessel-like function (rather than with an exponential[6,7]).

To determine the CP correction, we require $\arg(f_{21}/\eta_{+-})$; this was determined from $K_{\pi 2}$ yields

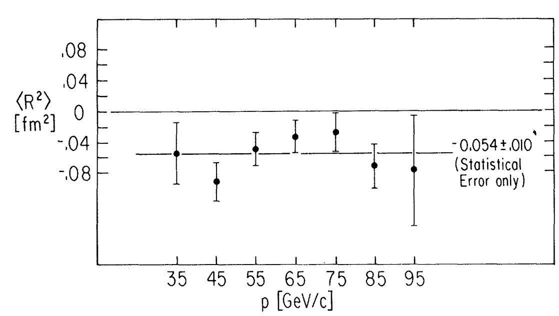

FIG. 3. $\langle R^2 \rangle$ vs momentum. Error flags indicate statistical errors. χ^2 for p-independent fit is 4.9 for six degrees of freedom.

in the two beams with and without the thin regenerator (fixing η_{+-} and imposing a soft constraint on σ_T[9]). We found $\arg f_{21}$ independent of p, with a mean $\langle \varphi \rangle = -122° \pm 1.8°$ while $(f - \bar{f})/k$ varied as p^{-n}, with $n = 0.65 \pm 0.02$. These values are "analytically compatible"[14] and the corresponding constraint was applied in the final fits.

$I_{\rm tr}(0)$ and $dI_{\rm diff}/dq^2(0)$ were then inputs to a fit which made the corrections for attenuation and CP nonconservation in computing $\langle R^2 \rangle$ for each p bin. When σ_T, $\langle \varphi \rangle$, and the f_{21} neutron shape

TABLE I. Upper part: all the correction factors (with errors) applied to the indicated numbers of thick coherent events and thin diffracted events per (100 MeV/c)^2 at $q^2 = 0$ in the extraction of the electron amplitude. Quoted errors are in many cases correlated. Lower part: contributions to the error in $\langle R^2 \rangle$ as a result of the uncertainties in the listed quantities required in the extraction of the electron amplitude.

A. Corrections (for 55 GeV/c bin)

	Thick Regenerator	Thin Regenerator	Category
Statistics	83874 ± 290	679.8 ± 5.5	1
$K_{\ell 3}$ background	0.9965 ± 0.0002	0.9300 ± 0.0035	2
Inelastics	0.9961 ± 0.0004	0.9840 ± 0.0016	3
Diffraction	0.9958 ± 0.0002	———	4
Regeneration in Anticounters, air	0.9948 ± 0.0010	.9968 ± 0.0006	5
Multiple Scattering $q^2 = 0$	———	0.9647 ± 0.0018	6
CP Correction	0.8818 ± 0.0030	1.0969 ± 0.0044	7
Attentuation	6.6204 ± 0.0428	1.4483 ± 0.0018	8
Acceptance	2.8095 ± 0.0075	2.8563 ± 0.0100	9

B. CONTRIBUTIONS OF VARIOUS UNCERTAINTIES TO ERROR IN $\langle R^2(K^0) \rangle$

	Variation	Contribution [fm²]	Category
Statistical	———	0.010	1,2,3,4,9
σ_T [a]	± 0.35%	0.012	8
f_{21} shape	± 0.04 fm	0.011	1
arg (f_{21})	± 1.2°	0.010	7
f_{22} shape [b]	± 0.2 fm	0.004	6
arg (f_{22})	c	0.002	6
Inelastic contribution at $q^2 = 0$	± 10%	0.004	3
$\lvert\eta_{+-}\rvert$	± 2%	< 0.001	7
Δm	± 1%	0.001	7
τ_S	± 0.3%	< 0.001	7
Result		-(0.054 ± 0.026)	

[a]From Ref. 9 and additional data; σ_T assumed constant for $50 < p < 100$ GeV/c.

[b]Parameter varied is rms nuclear radius; see Ref. 11.

[c]$\mathrm{Re}(f_{22})/\mathrm{Im}(f_{22}) \simeq 0.010$; error indicated is for change by ±0.005.

parameter are fixed at their best values, the result (see Fig. 3) is seen to be *negative* and *constant* ($\langle R^2\rangle_{av} = -0.054 \pm 0.010$ fm^2) and hence establishes regeneration from electrons. Table I lists the various corrections and sources of error; the upper part serves mainly for comparison with previous experiments,[6,7] while the lower part gives the systematic errors resulting from the fitting procedure. Allowing for the latter, we obtain $\langle R^2\rangle_{av} = -(0.054 \pm 0.026)$ fm^2. Our result is well within the range of recent theoretical predictions[3-5] and excludes with 98% probability the sign reported in Ref. 7.

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[7]F. Dydak *et al.*, Nucl. Phys. B102, 253 (1976).

[8]See, e.g., Ref. 7, whose notation we adopt for the reader's convenience.

[9]A. Gsponer *et al.*, "K_L-Nucleus Total Cross Sections between 30 and 150 GeV: Quantitative Evidence for Inelastic Screening" (to be published).

[10]A. Gsponer *et al.*, "Precise Coherent K_S Regeneration Amplitudes for C, Al, Cu, Sn, and Pb Nuclei from 20 to 140 GeV/c and Their Interpretation" (to be published).

[11]A full description of the details of this apparatus and the data analysis is given by W. R. Molzon, Ph.D. thesis, University of Chicago (unpublished), and to be published. See also A. Gsponer, Ph.D. thesis, Eidnössiches Technische Hochschule Zürich, Dissertation No. 6224, 1978 (unpublished), and J. Roehrig *et al.*, Phys. Rev. Lett. 38, 1116 (1977).

[12]R. H. Good *et al.*, Phys. Rev. 124, 1223 (1961).

[13]The first minimum has been observed previously by H. Foeth *et al.*, Phys. Lett. 31B, 544 (1970), at the same position as found here.

[14]That is, they fulfill the relationship $\arg(f_{21}) = -\frac{1}{2}\pi(2-n)$ corresponding to an amplitude satisfying dispersion relations.

ERRATA

DIRECT OBSERVATION OF ELASTIC AND INELASTIC PHOTON SCATTERING BY THE GIANT DIPOLE RESONANCE IN ^{60}Ni. T. J. Bowles, R. J. Holt, H. E. Jackson, R. M. Laszewski, A. M. Nathan, J. R. Specht, and R. Starr [Phys. Rev. Lett. 41, 1095 (1978)].

The following acknowledgment should be added: "A part of this research, including the development and operation of MUSL-II and its experimental areas, was supported by the National Science Foundation."

^{3}He AND ^{4}He FORM FACTORS AND THE DIMENSIONAL-SCALING QUARK MODEL. Benson T. Chertok [Phys. Rev. Lett. 41, 1155 (1978)].

On page 1158, the second and third lines below Eq. (6) should read "... (see Eq. 28 of Ref. 4) . . . ."

The sentence after Eq. (10) should read "... soft-core potentials.[4]"

In the third paragraph from the end of the Letter on page 1158, the references should be, "parton-model analyses.[4,12]"

DISAPPEARING CENTRAL PEAK IN PARAELECTRIC POTASSIUM DIHYDROGEN PHOSPHATE (KDP). Eric Courtens [Phys. Rev. Lett. 41, 1171 (1978)].

The correct caption of Fig. 2 should read as follows:

FIG. 2. A *typical* deviation plot. This plot applies to the experimental point indicated by the arrow in Fig. 1. For this spectrum the integrated number of photocounts is 2×10^5. The abscissa covers one-half free-spectral range, with the laser frequency at channel 0, and the Brillouin peak around channel 13 with 1.2×10^4 counts. The ordinate at channel i is $\sigma_i=(n_i-N_i)/\sqrt{n_i}$, where n_i is the measured and N_i the calculated photocount numbers. The χ^2 is $\sum\sigma_i^2/46=0.69$ for this particular fit.

K_S REGENERATION ON ELECTRONS FROM 30 TO 100 GeV/c: A MEASUREMENT OF THE K^0 CHARGE RADIUS. W. R. Molzon, J. Hoffnagle, J. Roehrig, V. L. Telegdi, B. Winstein, S. H. Aronson, G. J. Bock, D. Hedin, G. B. Thomson, and A. Gsponer [Phys. Rev. Lett. 41, 1213 (1978)].

The sentence beginning on line 5, column 1, p. 1213 should be delected and replaced with: "$\overline{K}^0$'s would be scattered with an amplitude $\bar{f}^e = -f^e$ since K^0 and $\overline{K}^0$ have conjugate charge distributions." Also, the sentence beginning on the third line, column 2, page 1214 should begin, "About 2×10^6 $K_{\pi2}$ candidates . . . ."

[9]R. J. Reid, Surf. Sci. 29, 623 (1972).
[10]L. F. Wagner, Z. Hussain, C. S. Fadley, and R. J. Baird, Solid State Commun. 21, 453 (1977).
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[13]*Tables of Interatomic Distances and Configurations in Molecules and Ions*, edited by L. E. Sutton (Chemical Society, London, 1958 and 1965 Supplement).
[14]*Crystal Data Determinative Tables*, edited by J. D. H. Donnay and H. M. Ondik (U. S. Department of Commerce, NSRDS, National Bureau of Standards and the Joint Committee on Powder Diffraction Standards, Washington, D. C., 1973), 3rd ed., Vol. II.

ERRATA

BÄCKLUND TRANSFORMATION FOR THE ERNST EQUATION OF GENERAL RELATIVITY. B. Kent Harrison [Phys. Rev. Lett. 41, 1197 (1978].

In Eq. (19), the $\frac{1}{2}$ on the right-hand side should be replaced by $\frac{1}{4}$. In Eq. (22), both appearances of $\frac{1}{2}$ should be replaced by $\frac{1}{4}$.

K_S REGENERATION ON ELECTRONS FROM 30 TO 100 GeV/c: A MEASUREMENT OF THE K^0 CHARGE RADIUS. W. R. Molzon, J. Hoffnagle, J. Roehrig, V. L. Telegdi, B. Winstein, S. H. Aronson, G. J. Bock, D. Hedin, G. B. Thomson, and A. Gsponer [Phys. Rev. Lett. 41, 1213,1523(E) (1978)].

The following should appear at the end of the Letter:

Note added.—See also the work of N. Isgur, Phys. Rev. D 17, 369 (1978), for a calculation of the mean-square K^0 charge radius.